**Eulerian Chains/Path**

* Must pass through **all** the **edges** of the graph once and only once
* Graph must contain exactly 2 odd degree vertices
* Graph must be connected
* Chain / Path will begin and end with odd degrees

**Example #1**

A

D

C

B

Step #1 – Determine the degree of each vertex

d(A) = 2 d(B) = 3 d(C) = 2 d(D) = 3



Exactly 2 odd degree vertices

Step #2 – Create chain / path that BEGINS and ENDS at the ODD DEGREE VERTICES

BADBCD or BCDABD or DABCDB or DCBDAB or etc….

**Example #2**

A

B

C

D

E

F

Step #1 - Determine the degree of each vertex

d(A) = 2 d(B) = 2 d(C) = 2 d(D) = 3 d(E) = 3 d(F) = 2

Exactly 2 odd degree vertices

Step #2 - Create chain / path that BEGINS and ENDS at the ODD DEGREE VERTICES

ECFDEABD

**Example #3**

F

E

D

C

B

A

Step #1 - Determine the degree of each vertex

d(A) = 2 d(B) = 2 d(C) = 2 d(D) = 2 d(E) = 1 d(F) = 1

Exactly 2 odd degree vertices

Step #2 - Create chain / path that BEGINS and ENDS at the ODD DEGREE VERTICES

IMPOSSIBLE  
Why? Because the graph is NOT connected  
You cannot reach points E or F from the other 4 vertices

**Example #4**

L

K

J

I

H

G

F

E

D

C

B

A

Step #1 - Determine the degree of each vertex

d(A) = 3 d(B) = 3 d(C) = 3 d(D) = 3 d(E) = 3 d(F) = 3

d(G) = 3 d(H) = 3 d(I) = 3 d(J) = 3 d(K) = 3 d(L) = 3

More than 2 ODD degrees

IMPOSSIBLE

**Eulerian Cycle / Circuits**

* A cycle / circuit that goes through **ALL** the **edges** of the graph once and only once
* Must **begin** and **end** at the same vertex
* Graph must connect **ALL EVEN** degrees
* Graph must be connected

**Example #1**

A

B

C

Step #1 – Determine the degree of each vertex

d(A) = 2 d(B) = 2 d(C) = 2

ALL even degrees….

Step #2 – Create a cycle that goes through each edge once and ends at the same vertex that you began.

ABCA  
or  
ACBA  
or  
CBAC  
or  
ETC….



Why could this example NOT be a Eulerian Chain / Path ???

**Example #2**

K

J

I

H

G

F

E

D

C

B

A

Step #1 – Determine the degree of each vertex

d(A) = 2 d(B) = 2 d(C) = 2 d(D) = 4 d(E) = 4 d(F) = 4

d(G) = 2 d(H) = 4 d(I) = 2 d(J) = 2 d(K) = 2

ALL even degrees….

Step #2 – Create a cycle that goes through each edge once and ends at the same vertex that you began.

ABDCEDHIJKFGHFEA

**Example #3**

Step #1 – Determine the degree of each vertex

d(A) = 2 d(B) = 3 d(C) =2

d(D) = 2 d(E) = 3

E

D

C

B

A

IMPOSSIBLE  
Not ALL vertices are EVEN

**Hamiltonian Chains / Paths**

* Chain / Path that goes through all the VERTICES of the graph once
* Graph must be connected

A

B

C

D

Chain / Path ABCD is Hamiltonian because it goes through ALL the VERTICES of the graph.

**Hamiltonian Cycles / Circuits**

* Cycle / Circuit that goes through all the VERTICES of the graph once
* Begins and Ends at the same vertex
* No degree 1 in the graph
* Graph must be connected

A

B

C

D

Cycle / Circuit DABCD is Hamiltonian because it goes through all the VERTICES of the graph once & begins and ends with the same vertex.

* Note: a sufficient but not necessary condition for a Hamiltonian Cycle is that each vertex of the graph is connected to at least half the other vertices, therefore, each vertex has a degree greater than or equal to *n/*2